Directions: Type your solutions into this document and be sure to show all steps for arriving at your solution. Just giving a final number may not receive full credit.

Problem 1

In the following question, the domain of **discourse** is a set of male patients in a clinical study. Define the following predicates:

* *P*(*x*) : *x* was given the placebo
* *D*(*x*) : *x* was given the medication
* *M*(*x*) : *x* had migraines

Translate each of the following statements into a logical expression. Then negate the expression by adding a negation operation to the beginning of the expression. Apply De Morgan’s law until each negation operation applies directly to a predicate and then translate the logical expression back into English.

*Sample question: Some patient was given the placebo and the medication.*

* ∃*x* (*P*(*x*) ∧ *D*(*x*))
* *Negation:* ¬∃*x* (*P*(*x*) ∧ *D*(*x*))
* *Applying De Morgan’s law:* ∀*x* (¬*P*(*x*) ∨ ¬*D*(*x*))
* *English: Every patient was either not given the placebo or not given the medication (or both).*

1. Every patient was given the medication or the placebo or both.

**Logical expression:** ∀*x*(*D*(*x*) ∨ *P*(*x*))

**Negation:** ¬∀*x*(*D*(*x*) ∨ *P*(*x*))

**Applying De Morgan’s law:** ∃*x*(¬*D*(*x*) ∧¬*P*(*x*)

**English:** Some patient was not given the medication and not given the placebo.

1. Every patient who took the placebo had migraines. (Hint: you will need toapply the conditional identity, *p* → *q* ≡¬*p* ∨ *q*.)

**Logical Expression:** ∀*x*(*P*(*x*) −→ *M*(*x*))

**Negation:** ¬∀*x*(*P*(*x*) −→ *M*(*x*))

**Conditional Identity:** ¬∀*x*(¬*P*(*x*) ∨ *M*(*x*))

**Applying De Morgan’s law:** ∃*x*(¬¬*P*(*x*) ∧¬*M*(*x*))

**Double negation law:** ∃*x*(*P*(*x*) ∧¬*M*(*x*))

**English:** Some patient was given the placebo but does not have a migraines.

1. There is a patient who had migraines and was given the placebo.

**Logical Expression:** ∃*x*(*M*(*x*) ∧ *P*(*x*))

**Negation:** ¬∃*x*(*M*(*x*) ∧ *P*(*x*))

**Applying De Morgan’s law:** ∀*x*(¬*M*(*x*) ∨¬*P*(*x*))

**English:** Every patient who does not have migraines was not given the placebo.

Problem 2

Use De Morgan’s law for quantified statements and the laws of propositional logic to show the following equivalences:

* 1. ¬∀*x* (*P*(*x*) ∧¬*Q*(*x*)) ≡ ∃*x* (¬*P*(*x*) ∨ *Q*(*x*))

∃*x*(¬*P*(*x*) ∨¬¬*Q*(*x*)) **De Morgan’s Law**

∃*x*(¬*P*(*x*) ∨ *Q*(*x*)) **Double negation law**

* 1. ¬∀*x* (¬*P*(*x*) → *Q*(*x*)) ≡ ∃*x* (¬*P*(*x*) ∧¬*Q*(*x*))

∃*x*(¬¬*P*(*x*) −→¬*Q*(*x*)) **De Morgan’s Law**

∃*x*(*P*(*x*) −→¬*Q*(*x*)) **Double negation law**

∃*x*(¬*P*(*x*) ∨¬*Q*(*x*)) **Conditional identity**



∀*x*(¬¬*P*(*x*) ∧¬(*Q*(*x*) ∧¬*R*(*x*)))**De Morgan’s law**

∀*x*(*P*(*x*) ∧¬(*Q*(*x*) ∧¬*R*(*x*)))**Double negation laws**

∀*x*(*P*(*x*) ∧ (¬*Q*(*x*) ∨¬¬*R*(*x*)))**De Morgan’s law**

∀*x*(*P*(*x*) ∧ (¬*Q*(*x*) ∨ *R*(*x*)))**Double negation laws**

Problem 3

The domain of **discourse** for this problem is a group of three people who are working on a project. To make notation easier, the people are numbered 1*,* 2*,* 3. The predicate *M*(*x, y*) indicates whether x has sent an email to *y*, so *M*(2*,* 3) is read “Person 2 has sent an email to person 3.” The table below shows the value of the predicate *M*(*x, y*) for each (*x, y*) pair. The truth value in row *x* and column *y* gives the truth value for *M*(*x, y*).

|  |  |  |  |
| --- | --- | --- | --- |
| *M* | 1 | 2 | 3 |
| 1 | *T* | *T* | *T* |
| 2 | *T* | *F* | *T* |
| 3 | *T* | *T* | *F* |

**Determine if the quantified statement is true or false. Justify your answer.**

1. ∀*x*∀*y* (*x* 6= *y*) → *M*(*x, y*))

(true) No matter the person, when not including themselves, they had send the email to everyone else.

1. ∀*x*∃*y* ¬*M*(*x, y*)

(false) The table is negated, leaving person 1 not sending an email to anyone.

1. ∃*x*∀*y M*(*x, y*)

(true) Person 1 has sent the email to everyone.

Problem 4

Translate each of the following English statements into logical expressions. The domain of **discourse** is the set of all real numbers.

1. The reciprocal of every positive number less than one is greater than one.



1. There is no smallest number.

¬∃*x*∀*y*(*x* ≤ *y*)

1. Every number other than 0 has a multiplicative inverse.

∀*x*∃*y*(*x* ≠ 0 → (*xy* = 1))

Problem 5 The sets *A*, *B*, and *C* are defined as follows:

* 1. = *tall,grande,venti*
  2. = *foam,no* − *foam C* = *non* − *fat,whole*

Use the definitions for *A*, *B*, and *C* to answer the questions. Express the elements using *n*-tuple notation, not string notation.

1. Write an element from the set *A* × *B* × *C*.

(grande,foam,whole)

1. Write an element from the set *B* × *A* × *C*.

(no-foam,tall,non-fat)

1. Write the set *B* × *C* using roster notation.

*B*×*C* = {(*foam,non*−*fat*)*,*(*foam,whole*)*,*(*no*−*foam,non*−*fat*)*,*(*no*− *foam,whole*)}